# BIG SHIFTS IN MATHEMATICS CONTENT K-8

## Kindergarten

The sentence from the Common Core State Standards for Mathematics (CCSSM) that points to the biggest shift for kindergarten teachers is that there are two critical areas of study: Number/Operations and geometric reasoning. The following quote from the CCSSM clearly indicates the focus for Kindergarten: "More learning time in kindergarten should be devoted to number than any other topic." In order to devote this time, some things that were in the Arkansas Mathematics Frameworks for Kindergarten are not found in the CCSSM.

Although Counting and Cardinality are still part of Kindergarten there is a shift toward understanding the concept of "ten" as a unit with a real focus on understanding the number 11-19 as one group of ten and some toward the next group of ten. (K.NBT.1) This standard does not limit the teaching of number to 19. In fact, another standard (K.CC.1) requires that students "count to 100 by ones and tens." While teaching students to count the concept of "ten" as a unit can continue to develop.

The other major shift is the expectation that students will understand addition and subtraction as actions related to situations. This is indicated in the description of Operations and Algebraic Thinking domain and clarified in "Table 1: Common addition and subtraction situations" found on page 88 of the CCSSM.

In geometry, students are expected to move beyond vocabulary and compare two- and three-dimensional shapes based on attributes of the shapes.

## First Grade

In first grade, two of the four foci deal exclusively with number concepts. Thus as in Kindergarten, more of the instructional time in first grade will be spent on place value and addition and subtraction concepts than on other topics.

Student will not just learn how to add and subtract but to identify situations in which addition and subtraction are appropriate. These situations are summarized in "Table 1: Common addition and subtraction situations" found on page 88 of the CCSSM.

Another big shift in first grade is the intent to have students understand two-digit numbers and the use of the concept of "ten" as a unit to record, compare and compute with these numbers.

Perhaps the biggest shift for first grade teachers will be the expectation that student "understand and apply properties of operations and the relationship between addition and subtraction" as a way of making sense of number and operations. This is a way of thinking and generalizing ideas that later apply to larger and rational

numbers. Under the Arkansas Frameworks, these ideas were not given special emphasis or specifically connected to number and operations.

Another shift is related to linear measurement. The foundation for critical understanding of linear measurement and the development of tools is established in first grade. Students are to make sense of the underpinnings of the principles of measure (identical units, iteration, transitivity and no gaps/overlaps) that lead to the development of measurement tools.

Geometry is another shift for first grade. Students need to build an understanding of properties of shapes and their relationships to each other. While fractions are not addressed in number and operations, exploring and making sense of part-whole relationships through equal sharing experiences is embedded in the continued exploration of shape and form.

**Second Grade**

As in first grade, two of the four foci for grade two deal exclusively with number concepts. This will require a significant increase in the amount of instructional time spent on these topics.

Certainly the expectation that "all" students can fluently add and subtract within 20 will produce a shift for teachers at this grade. Teachers will need to understand that fluency does not develop without understanding and appropriate practice. The research on how fluency develops needs to be communicated to teachers. This includes building an "understanding and applying properties of operations and the relationship between addition and subtraction" as a way of making sense of number and facts. This builds on the generalized thinking from first grade. If what researchers have found is implemented this shift will not take teacher back to ineffective timed drill.

Another significant shift for second grade teachers will help students explore and understand how grouping by tens extends to hundreds once ten groups of ten have been formed. The CCSSM intend that students not just see patterns in the way numbers are recorded, but understand the structure of the recording system and expansion of understanding and application properties of operations to 1000

i.e. $16 \times 10 = (10 \times 10) + (6 \times 10)$ or $50 \times 10 = 5 \times (10 \times 10)$ or

$1000 = (10 \times 10) \times 10 = 10 \times (10 \times 10)$

The biggest shift in number and operations for second grade is in the area of adding and subtracting within 100. The CCSSM clearly state that students use the "traditional" algorithm for addition and subtraction in grade ***four***. Many teachers rely exclusively on the traditional algorithm for performing these operations but the CCSSM is very clear that in second grade students "fluently add and subtract within 100 using ***strategies*** **based on place value, properties of operations and the relationship between addition and subtraction**." (2.NBT.5)

While not clearly defined, linear measurement brings a significant shift from Arkansas Frameworks. Students will develop an understanding for the need for standard units and **translate the underpinnings** from first grade **to the development of tools** for linear measurement (inches, feet, yards, centimeters and meters). Without making these connections, all will be for naught. Partial units for linear measurement are not included in second grade, but it is critical to note that after second grade, linear measurement does not appear again in the CCSS. There is an indirect route into fractions on a number line under Number and operations – Fractions in 3rd grade. If students are to learn linear measurement, districts will need to decide who is truly responsible for this learning. One of the questions facing districts will be: Does second grade take the full accountability for developing the linear measurement with understanding (including partial units) or pass the completion of the ideas to third grade?

**Third Grade**

As in first and second grade, two of the four foci for grade three deal exclusively with number concepts with a heavy emphasis on developing a deep understanding of fractions. This will require a significant increase in the amount of instructional time spent on these topics.

Perhaps the one of the biggest shifts for third grade teachers will be the expectation that students understand properties of operations beyond inverse operation for addition and subtraction to the relationship between multiplication and division and apply these ideas a way of making sense of number and operations. The generalization of these ideas should be used to develop understanding and support student thinking about facts, the four operations with multi-digit numbers, base 10 concepts and fractions. Under the Arkansas Frameworks these ideas were not given special emphasis or specifically connected to number and operations.

Certainly the expectation that "all" students can fluently multiply and divide within 100 in the different problem situations will produce another major shift for teachers at this grade. The different problem situations are found in "Table 2: Common multiplication and division situations" on page 89 of the CCSSM. The CCSSM stress that students should learn facts based on "understanding and the application properties of operations and the relationship between multiplication and division." Teachers need to understand that fluency does not mean the development of facts without understanding and appropriate practice. If what researchers have found is implemented, this shift should not take teacher back to ineffective timed drill.

In second grade, students are expected to make sense of number through 1000 and in fourth grade students are expected to generalize the whole number base-ten numeration system. Third grade students are expected to round and solve problems within 1000. If students are expected to extend and generalize their thinking from second grade to fourth, third grade will need to continue to strengthen the work in 2nd grade and build on these ideas if they are going to be in the position to generalize in fourth.

Another huge shift is towards an in-depth conceptual understanding of fractions in contextualized situations both in number and operations and as part of geometric reasoning through the applications of properties of operations. The thinking goes far beyond the simple identification of fractional parts using fraction models.

Third grade will focus also efforts in measurement to exploring and making sense of area by applying the basic constructs of equal sized units, iteration, no gaps and overlaps. Students are expected use their understanding of area to generalize the formula for finding the area of a rectangle. Again while not explicit in the common core standards, teachers will have to provide students with experiences to help them develop spatial structure (seeing a row iterated across the area or the column iterated across the area). Michael Battista and others have done research on "Students' Spatial Structuring of 2D Arrays of Squares " describing learning progressions of students' ability to structure space. (Battista, M. T., Clements, D. H., Arnoff, J., Battista, K., & Borrow, C. V. A. (1998 November). Students' spatial structuring of 2D arrays of squares. *Journal for Research in Mathematics Education 29*(5), 503-532.)

**Fourth Grade**

Fourth grade has three areas of foci, but two of those still deal with number concepts. According to the Common Core document, three critical areas of focus are: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties.

Students will generalize their understanding of whole number place value and the relative sizes of numbers in each place based on base-ten concepts and properties of operations {i.e. 10,000 – 10 x (10 x (10x 10))}. The emphasis on developing understanding *and fluency* with multi-digit multiplication and developing understanding of dividing to find quotients involving multi-digit dividends by applying knowledge of base-ten numeration and properties of these operations. This will require a greater amount of time spent in these areas and will be dependent on the understandings and generalizations students developed in the previous grades.

A big shift will be the emphasis on developing student's ability to explain their reasoning and use multiple methods of solving problems. Students are expected to learn the concepts, see them relationally (based on the properties of operations) and invent strategies for solving problems. This will require a great deal of class time to develop the knowledge, understanding, and skills needed to achieve this

goal. Teachers may also require professional development in questioning skills and procedures that allow students to develop these skills.

One of the biggest shifts is the emphasis on fractions and the depth to which they are taught. A great deal of the CCSSM fraction piece for fourth grade was taught in the fifth and sixth grade Arkansas frameworks. This may challenge the teachers as well as the students. Students will develop understanding of fraction equivalence and operations with fractions; recognize that unlike fractions can be equal; develop methods for generating and recognizing equivalent fractions. Students will build on previous understandings of how fractions are built; compose and decompose fractions into unit fractions; and multiply a fraction by a whole number based on properties of operations. Students will also show understanding of decimal notation for fractions and compare decimal fractions,

The measurement and data portion will require students to solve measurement problems and convert from a larger unit to a smaller unit. Students will also represent and interpret data and understand concepts of angle and measure angles.

The shift in geometry appears small at first glance, but when you look at the depth to which it must be taught, the shift is quite large. The focus in geometry is on two-dimensional figures, lines, and angles, but many concepts have moved down from fifth grade and above. Students will describe, analyze, and classify two-dimensional figures. Through building, drawing, and analyzing these shapes, students will gain a deeper understanding of properties of two-dimensional objects and how to use them to solve problems related to symmetry.

**Fifth Grade**

The CCSSM document states that the primary focus for fifth grade students should be: (1) the *development of fluency* with addition and subtraction of fractions and *developing understanding* of the multiplication and division of fractions in limited cases; (2) the extension of division to 2-digit divisors; integrating decimal fractions into the place value system; developing understanding of operations with decimals to hundredths; and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

Students will use equivalent fractions as a strategy to add and subtract fractions with unlike denominators. They will solve relevant word problems that involve addition and subtraction of fractions referring to the same whole using visual models or equations.

Fifth grade students will develop an understanding of why division procedures work and finalize fluency with multi-digit addition, subtraction, multiplication, and division. They will apply their understandings of decimals and decimal notation and will be able to make reasonable estimates of computations.

Several items dealing with patterns, relations and functions, as well as rational numbers and use of technological tools found in the sixth through eighth grade

Arkansas frameworks have shifted to fifth grade in CCSSM. Students must identify and extend patterns in real world situations and be able to invent strategies to solve problems using function tables and linear equations.

Measurement focuses on the conversion of like units within a given measurement system, representing and interpreting data, and understanding the concepts of volume and relating it to multiplication and to addition. These skills are all used in solving multi-step, real world problems. The strategies for finding volume are shifting from third and fourth grade to fifth grade, which means students will not have that foundation previously laid in those earlier grades.

Fifth graders will graph points on the coordinate plane in order to solve real-world and mathematical problems. They will also classify two-dimensional figures into categories based on their properties. The rigor and relevance of the work should be stressed.

**Sixth Grade**

Teachers will find that much of the content that was beginning and developing in grade 6 in the Arkansas Frameworks is reaching a culminating or fluency standard in grade 6 in the CCSSM. Sixth grade contains the expectation for fluency with multi-digit division (expectations for fluency in the other three operations have occurred in earlier grades), and with all four operations with decimals. The last operation for fractions, division, is begun in fifth grade and continued into sixth grade.

The change in domains in grade 6 indicates that sixth grade is a pivotal point when the focus begins to shift from number and operations (K-5) to the underpinning of algebra.

Proportional reasoning emerges as a major topic in grade 6. The CCSSM recognizes proportional reasoning as one of the more powerful types of reasoning needed by adults, and defines it as a focus on instruction in the middle grades. In grade 6, students are asked to connect ratio and rate to whole number multiplication and division and use ratio and rate to solve problems. Their work will include the use of equivalent ratios, unit rates, and percent.

Students in grade 6 will encounter negative numbers (additive inverses of all the kinds of numbers they have already studied) as a final element needed to complete the rational number system. They will consider the relative locations of various numbers on a number line. Having a firm understanding of the entire rational number system will be vital for success as students move into seventh grade. Positive and negative numbers will be used to locate points in a coordinate plane as well.

Also, sixth graders will learn about data distributions and statistics. Rather than just learning to calculate convention measures of center, they will examine distributions of numerical data, learn about and understand both central tendency and statistical variability, and summarize distributions using appropriate statistics. They will not

only learn to calculate statistics to measure center, they will explore the vulnerabilities of these measures to characteristics of the data (i.e., that the mean can be skewed by outliers). They will also explore statistics that measure variability and consider their uses as well. These are topics that have previously been approached after grade 6.

**Seventh Grade**

Teachers in seventh grade may be among those who feel fewer shifts in actual content coming into the grade and more of a shift toward being able to delve deeper into fewer major topics of emphasis. The key word for 7th grade teachers is **focus**.

Examining proportional relationships in various forms (including equation, graph, table) and applying them to solve problems (including with scale drawings) have been in the Arkansas seventh grade curriculum in the past and are even more important in the CCSSM. An examination of these new standards will reveal the expectation that seventh graders develop a strong, flexible understanding of proportions and their applications. (7.RP.1-3; 7.G.1)

Similarly, extending operations to the full rational number system (including integers and negative fractions), working with and solving linear equations, solving problems involving area, surface area, and volume, and drawing inferences about populations based on samples have all been included in the Arkansas 7th grade curriculum in the past. The defining difference is the focus on these topics, which reflects the intent that students spend larger portions of time developing conceptual understanding and applying the related skills to solve problems.

In order to allow more time for these important focus topics, other time-consuming topics have been shifted out of 7th grade mathematics, such as: graphing in the coordinate plane, understanding integers and negative fractions as part of the rational number system, developing the concepts of surface area and volume, and using scientific notation. Some topics lingering from elementary have also been shifted out, such as: elapsed time, linear measure, and finding area of simple polygons.

**Eight Grade**

Two of the three focus areas for Eight Grade deal with algebraic concepts. In Eight Grade students will learn many of the algebra concepts that are currently found in the Arkansas Mathematics Frameworks for Algebra I.

Eight grade students will expand their understanding of number to include irrational numbers. This will require that these students work with radicals and transcendental numbers like pi. This includes using roots to solve equations of the $x^n=p$, where n is a natural number and p is a positive rational number. The properties of integer exponents have moved from Algebra I to Eighth Grade in the CCSSM. Students are required to fluently generate equivalent expressions using the properties of integer exponents. The use of integer exponents includes fluency in

***SOLVING*** (not just converting) problems involving scientific notation (including multiplication and division). This is also moving from Algebra I to Eighth Grade.

Eighth Grade students are required by CCSSM to understand the connections between proportional relationships, lines and linear equations. Almost the entire linear function section of our current Algebra I content is shifting to the Eighth Grade in the CCSSM. This includes: effects of parameter changes, slope, y intercepts on graphs of linear functions; calculating the slope using various methods (given differing information); writing equations of linear functions (presented in various forms) given different entry points.

The CCSSM require Eight Graders to analyze and solve (using a variety of methods) linear equations and systems linear equations in two variables. This includes using linear equations and systems of two equations in two variable to solve application (real-world) problems.

Students learn about functions in Eighth Grade under the CCSSM. Students are required to define, evaluate and compare functions. Students will need to distinguish between functions and non-functions by inspecting graphs, ordered pairs, mapping diagrams, and/or tables of data. Students will need to determine the domain and range from an algebraic expression, graphs, set of ordered pairs, or table of data. Finally, the student needs compare rates of change in different types of functions.

The final big shifts are in the geometry area. Student will now work to understand congruence and similarity using physical models, transparencies or geometry software. This is almost completely new at this grade level. Under the Arkansas Frameworks these concepts were done in Geometry Class. Also, in the geometry domain CCSSM requires students in Eighth Grade to explain a proof of the Pythagorean theorem. The Pythagorean theorem has been in Eighth Grade but the requirement to prove it has moved from High School Geometry Class.